Maths Progression Map – Whybridge Junior School				
	Year 3	**Year 4**	**Year 5**	**Year 6**
Counting	• count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number.	• count in multiples of 6, 7, 9, 25 and 1000 • find 1000 more or less than a given number • count backwards through zero to include negative numbers	• count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 • interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero	• use negative numbers in context, and calculate intervals across zero
Place Value	• recognise the place value of each digit in a three-digit number • compare and order numbers up to 1000	• recognise the place value of each digit in a four-digit number • order and compare numbers beyond 1000 • round any number to the nearest 10, 100 or 1000	• read, write, order and compare numbers up to 1 000 000 and determine the value of each digit • round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000	• read, write, order and compare numbers up to 10 000 000 and determine the value of each digit • round any whole number to a required degree of accuracy
Representing number	• identify, represent and estimate numbers using different representations • read and write numbers up to 1000 in numerals and in words	• identify, represent and estimate numbers using different representations • read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value	• read Roman numerals to 1000 (M) and recognise years written in Roman numerals • recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})	
Mental +/-	• add and subtract numbers mentally, including: HTU+U, HTU+T and HTU+H		• add and subtract numbers mentally with increasingly large numbers	• perform mental calculations, including with mixed operations and large numbers
Written +/-	• add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	• add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	• add and subtract whole numbers with more than 4 digits, including using formal written methods	
Problems +/-	• estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction	• estimate and use inverse operations to check answers to a calculation • solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	• use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	

Number facts (x/÷)	• recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	• recall multiplication and division facts for multiplication tables up to 12 × 12	• identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers • establish whether a number up to 100 is prime and recall prime numbers up to 19	• identify common factors, common multiples and prime numbers
Mental (x/÷)	• write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods	• use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations	• multiply and divide numbers mentally drawing upon known facts • multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	• perform mental calculations, including with mixed operations and large numbers
Written (x/÷)	• Progress to formal written methods calculations as above	• multiply two-digit and three-digit numbers by a one-digit number using formal written layout	• multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context • divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to context
Problems (x/÷)	• solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.	• solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects	• solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates	• use their knowledge of the order of operations to carry out calculations involving the four operations • solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers

				to calculations and determine, in the context of a problem, an appropriate degree of accuracy
Recognising fractions	• count up and down in tenths; • recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10	• count up and down in hundredths; • recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.	• recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number	
Comparing fractions	• compare and order unit fractions, and fractions with the same denominators • recognise and show, using diagrams, equivalent fractions with small denominators	• recognise and show, using diagrams, families of common equivalent fractions	• compare and order fractions whose denominators are all multiples of the same number • identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths	• use common factors to simplify fractions • use common multiples to express fractions in the same denomination • compare and order fractions, including fractions > 1
Finding fractions of quantities	• recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators • recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators	• solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number		
Calculating with fractions	• add and subtract fractions with the same denominator within one whole [for example, 5/7 + 1/7 = 6/7]	• add and subtract fractions with the same denominator	• add and subtract fractions with the same denominator and denominators that are multiples of the same number • multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	• add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions • multiply simple pairs of proper fractions, writing the answer in its simplest form • divide proper fractions by whole numbers
Decimals as fractional amounts		• recognise and write decimal equivalents of any number of tenths or hundredths • recognise and write decimal equivalents to ¼, ½ and ¾ • find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths	• read and write decimal numbers as fractions	• associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction • identify the value of each digit in numbers given to three decimal places

Ordering decimals		• round decimals with one decimal place to the nearest whole number • compare numbers with the same number of decimal places up to two decimal places	• recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents • round decimals with two decimal places to the nearest whole number and to one decimal place • read, write, order and compare numbers with up to three decimal places	
Calculating with decimals				• multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places • multiply one-digit number with up to two decimal places by whole numbers • use written division methods in cases where the answer has up to two decimal places
Percentages			• recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal	• solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison
Fraction problems	• solve problems using all fraction knowledge	• solve simple measure and money problems involving fractions and decimals to two decimal places	• solve problems involving number up to three decimal places • solve problems which require knowing percentage and decimal equivalents of ½ , ¼ , 1/5 , 2/5 , 4/5 and those fractions with a denominator of a multiple of 10 or 25	• solve problems which require answers to be rounded to specified degrees of accuracy • recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.
Ratio & Proportion				• solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts • solve problems involving similar shapes where the scale factor is known or can be found • solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

Algebra				• use simple formulae • generate and describe linear number sequences • express missing number problems algebraically • find pairs of numbers that satisfy an equation with two unknowns • enumerate possibilities of combinations of two variables.
Measures	• measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)	• Convert between different units of measure • estimate, compare and calculate different measures, including money in pounds and pence	• convert between different units of metric measure • understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints • estimate volume and capacity	• solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate • use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places • convert between miles and kilometres
Mensuration	• measure the perimeter of simple 2-D shapes	• measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres • find the area of rectilinear shapes by counting squares	• measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres • calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes	• recognise that shapes with the same areas can have different perimeters and vice versa • recognise when it is possible to use formulae for area and volume of shapes • calculate the area of parallelograms and triangles • calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm3) and cubic metres (m3), and extending to other units.
Money	• add and subtract amounts of money to give change, using both £ and p in practical contexts		• use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling	

Time	• tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks • estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight • know the number of seconds in a minute and the number of days in each month, year and leap year compare durations of events	• Convert between different units of measure (e.g. Hours to minutes) • read, write and convert time between analogue and digital 12- and 24-hour clocks • solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days	• solve problems involving converting between units of time	
Shape vocabulary	• identify horizontal and vertical lines and pairs of perpendicular and parallel lines			• illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
Properties of 2-d shape	• draw 2-D shapes	• compare and classify geometric shapes, including quadrilaterals and triangles, based on properties and sizes • identify lines of symmetry in 2-D shapes presented in different orientations • complete a simple symmetric figure with respect to a specific line of symmetry.	• use the properties of rectangles to deduce related facts and find missing lengths and angles • distinguish between regular and irregular polygons based on reasoning about equal sides and angles.	• draw 2-D shapes using given dimensions and angles compare and classify geometric shapes based on their properties and sizes
Properties of 3-d shape	• make 3-D shapes using modelling materials • recognise 3-D shapes in different orientations and describe them		• identify 3-D shapes, including cubes and other cuboids, from 2-D representations	• recognise, describe and build simple 3-D shapes, including making nets • find unknown angles in any triangles, quadrilaterals, and regular polygons
Angles	• recognise angles as a property of shape or a description of a turn • identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn • identify whether angles are greater or less than right angle	• identify acute and obtuse angles and compare and order angles up to two right angles by size	• know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles • draw given angles, and measure them in degrees (°) • identify angles at a point and one whole turn (total 360°); at a point on a straight line and ½ a turn (total 180°) • identify other multiples of 90°	• recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles

Position & Direction		• describe positions on a 2-D grid as coordinates in the first quadrant • describe movements between positions as translations of a given unit to the left/right and up/down • plot specified points and draw sides to complete a given polygon	• identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed	• describe positions on the full coordinate grid (all four quadrants) • draw and translate simple shapes on the coordinate plane, and reflect them in the axes.
Interpreting data	• interpret and present data using bar charts, pictograms and tables	• interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs	• complete, read and interpret information in tables, including timetables	• interpret and construct pie charts and line graphs calculate and interpret the mean as an average
Extract info from data	• solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables	• solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs	• solve comparison, sum and difference problems using information presented in a line graph	• use pie charts and line graphs to solve problems

Whybridge Junior School

Written Methods in Mathematics

At Whybridge Junior School, the children are introduced to the processes of calculation through practical, oral and mental activities. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful. (Guidance on Calculation, Primary Mathematics Framework, National Strategies)

At Whybridge Junior School, our methods of calculation focus on stages the children are at in their learning, rather than by year group. That way, we are differentiating for the needs of our children in our class. It is important to follow this policy as a rule as each child moves up through the school to ensure consistent teaching approaches in the mathematics lessons.

The overall aim is that when children leave primary school they:

- have a secure knowledge of number facts and a good understanding of the four operations;
- are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;
- make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally.

The objectives in the revised Framework show the progression in children's use of written methods of calculation from Year 2 to Year 6

Year 2 • Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts • Use the symbols +, -, × , ÷ and = to record and interpret number sentences involving all four operations; calculate the value of an unknown in a number sentence (e.g. ? ÷ 2 = 6, 30 - ? = 24) • Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.
Year 3

- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Year 4

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate-solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Year 5

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Year 6

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division

Addition

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 4.

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;

- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

<table>
<tr><th>Method</th><th>Example</th></tr>
<tr><td colspan="2">Stage 1: The empty number line</td></tr>
<tr><td>

- The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.
- The empty number line helps to record the steps on the way to calculating the total.

</td><td>
Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

8 + 7 = 15

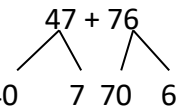

48 + 36 = 84


or:


</td></tr>
<tr><td colspan="2">Stage 2: Partitioning</td></tr>
<tr><td>

- The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums.
- Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

</td><td>
Record steps in addition using partitioning:



Partitioned numbers are then written under one another:

$$\begin{array}{rcl} 47 & = & 40 + 7 \\ +\,76 & & 70 + 6 \\ \hline & & 110 + 13 = 123 \end{array}$$
</td></tr>
<tr><td colspan="2">Stage 3: Expanded method in columns</td></tr>
<tr><td>

- The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how

</td><td>
Write the numbers in columns.

Adding the ones first:
$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 13 \\ 110 \\ \hline 123 \end{array}$$
</td></tr>
</table>

Method	Example
secure the children are in their recall of number facts and in their understanding of place value.	
Stage 4: Column method	
• In this method, recording is reduced further. Carry digits are recorded below the line or next to the numbers being added, using the words 'carry ten' or 'carry one hundred', not 'carry one'. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	47 + 76 123 11 Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable. 4.7 + 27.6 32.3 11

Subtraction

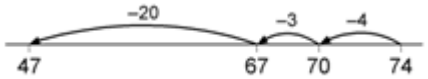
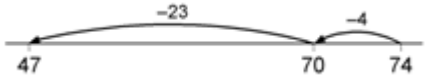
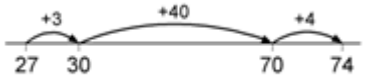
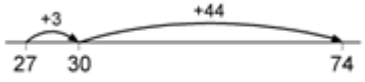
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

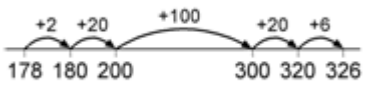
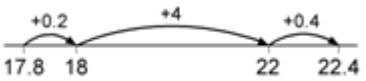
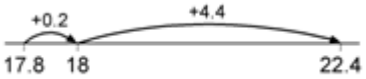
To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20
- subtract multiples of 10 (such as 160 - 70) using the related subtraction fact, f16 - 7, and their knowledge of place value
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Method	Example
Stage 1: Using the empty number line	

Method	Example
• The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 - 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. • The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. • With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 - 12, 86 - 77 or 43 - 28. The notes below give more detail on the counting-up method using an empty number line.	Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. 15 - 7 = 8  74 - 27 = 47 worked by counting back:  The steps may be recorded in a different order:  or combined: 
The counting-up method	
• The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ? = 74 mentally.	 74 − 27 3 40 4 47 Or:  74 − 27 3 44 47

Method	Example
• With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ? = 200 and 200 + ? = 326 mentally. • The most compact form of recording remains reasonably efficient.	 326 −178 2 20 100 26 148 Or:  326 −178 22 126 148
• The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed. • This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.	 22.4 −17.8 0.2 4.0 0.4 4.6 Or:  22.4 −17.8 0.2 →18 4.4 →22.4 4.6
Stage 2: Partitioning	
• Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally.	Subtraction can be recorded using partitioning: 74 – 27 74 - 20 - 7 = 54 - 7 = 47
Stage 3: Expanded layout, leading to column method	

<table>
<tr><th>Method</th><th>Example</th></tr>
<tr><td>

- Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.
- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

</td><td>

Partitioned numbers are then written under one another:

Example: 78 – 27

$$\begin{array}{r} 70 + 8 \\ -\ 20 + 7 \\ \hline 50 + 1 \end{array} = 51$$

Example: 74 - 27

$$\begin{array}{r} 70 + 4 \\ -\ 20 + 7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{60}{\cancel{70}} + \overset{14}{\cancel{4}} \\ -\ 20 + 7 \\ \hline 40 + 7 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

Example: 741 - 367

$$\begin{array}{r} 700 + 40 + 1 \\ -\ 300 + 60 + 7 \\ \hline \end{array} \qquad \begin{array}{r} \overset{600}{\cancel{700}} + \overset{130}{\cancel{40}} + \overset{11}{\cancel{1}} \\ -\ 300 + 60 + 7 \\ \hline 300 + 70 + 4 \end{array} \qquad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

</td></tr>
<tr><td colspan="2">The expanded method for three-digit numbers</td></tr>
<tr><td></td><td>

Example: 503 – 278, dealing with zeros when adjusting

$$\begin{array}{r} 500 + 0 + 3 \\ -\ 200 + 70 + 8 \\ \hline \end{array} \qquad \begin{array}{r} 400 + 90 + 13 \\ -\ 200 + 70 + 8 \\ \hline 200 + 20 + 5 \end{array}$$

$$\begin{array}{r} \overset{4}{\cancel{5}}\ \overset{9}{\cancel{0}}\ \overset{13}{\cancel{3}} \\ -\ 2\ 7\ 8 \\ \hline 2\ 2\ 5 \end{array}$$

</td></tr>
</table>

Multiplication

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

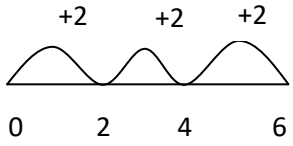
To multiply successfully, children need to be able to:

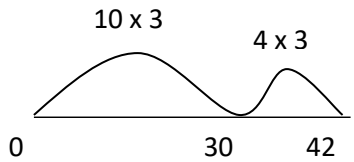
- recall all multiplication facts to 10 × 10
- partition number into multiples of one hundred, ten and one
- work out products such as 70 × 5, 70 × 50, 700 × 5 or700 × 50 using the related fact 7 × 5 and their knowledge of place value
- add two or more single-digit numbers mentally
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication. In Year 3, children use apparatus, arrays and number lines jumps for those having difficulty.

2 x 3 as an array

OOO or OO
OOO OO
OO



Method	Example
Stage 1: Mental multiplication using partitioning	
Mental methods for multiplying TO × O can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.	Informal recording in Year 4 might be: 43 40 + 3 × 6 240 + 18 = 258 Also record mental multiplication using partitioning: 14 x 3 10 x 3 = 30 4 x 3 = 12 30 + 12 = 42 
Stage 2: The grid method	
• As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.	38 × 7 = 210 + 56 = 266 × | 7 30 | 210 8 | 56 266 The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.
Stage 3: Expanded short multiplication	
• The next step is to represent the method of recording in a column format, but showing	38 X 7 56

Method	Example
the working. Draw attention to the links with the grid method above. • Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven', although the relationship 3 × 7 should be stressed. • Most children should be able to use this expanded method for TO × O by the end of Year 4.	210 266
Stage 4: Short multiplication	
• The recording is reduced further, with carry digits recorded below the line. • If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.	38 × 7 266 5 This can be used for decimals too however we teach children to remove the decimal point and make a note of how many decimal places were in the answer to begin with. Children then ensure there is the same number of decimal places left in the answer. 3.8 X 7 26.6 5
Stage 5: Long multiplication	
• Extend to TO × TO, asking children to estimate first. • This is the same method as short multiplication however with another digit • Children are taught to describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 124 x 26 is 'four multiplied by six', and then it is 'twenty multiplied by six' not 2 times 6, although the relationship 2 x 6 should be stressed. • Children are then taught to remember to put their 'place holder' in the ones column when moving onto multiplying by the second digit 'twenty' which the children refer to as '2' Children are taught this as they are now multiplying with a number in	124 × 26 becomes 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1 Answer: 3224

Method	Example
the tens column so the place holder ensures the answer will be ten times bigger.	
• Reduce the recording, showing the links to the grid method above.	56 × 27 is approximately 60 × 30 = 1800. 56 × 27 1000 50 × 20 = 1000 120 6 × 20 = 120 350 50 × 7 = 350 42 6 × 7 = 42 1512 1
• Reduce the recording further. • The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally. • The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5. • This can be extended to HTU x TU etc.	56 × 27 is approximately 60 × 30 = 1800. 56 × 27 1120 56 × 20 392 56 × 7 1512 1

Division

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. These notes show the stages in building up to long division through Years 4 to 6 - first long division TO ÷ O, extending to HTO ÷ O, then HTO ÷ TO, and then short division HTO ÷ O.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division - for example in 18 ÷ 3 = 6,the 18 is the dividend, the 3 is the divisor and the 6 is the quotient
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value
- know how to find a remainder working mentally - for example, find the remainder when 48 is divided by 5
- understand and use multiplication and division as inverse operations.

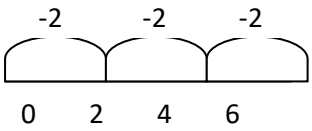
Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

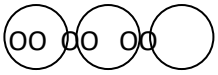
To carry out written methods of division successful, children also need to be able to:

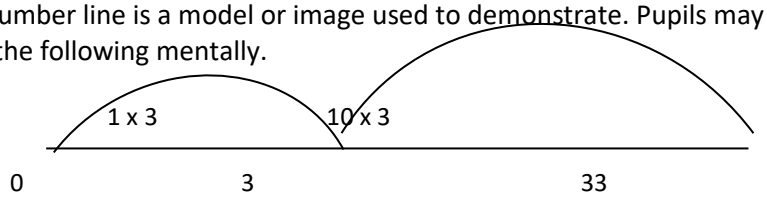
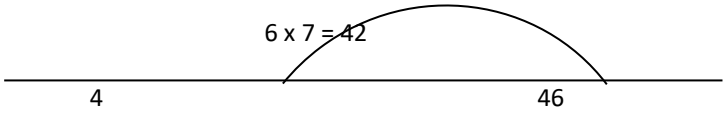
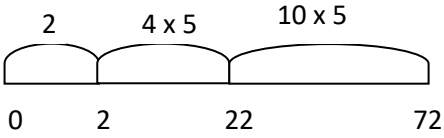
- understand division as repeated subtraction
- estimate how many times one number divides into another - for example, how many sixes there are in 47, or how many 23s there are in 92

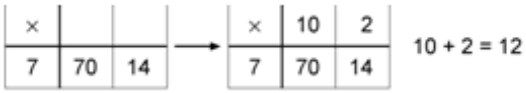
- multiply a two-digit number by a single-digit number mentally
- subtract numbers using the column method.

In Year 3, children may start with repeated subtraction, sharing/grouping using apparatus, using the inverse, using halving as the opposite of doubling, using language such as: how many 3s in 15? How many 2s in 6?





Method	Example
Stage 1: Mental division using partitioning	
Strategies, Models and Images The empty number line is a model or image used to demonstrate. Pupils may use jottings or calculate the following mentally. **33 ÷ 3**  Using knowledge of multiplication and division facts to find remainders e.g. 46 ÷ 6 = 7 r 4  72 ÷ 5 = Can we subtract 10 lots of 5? How many other lots of 5 can we subtract?  	
• Mental methods for dividing TO ÷ O can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient. • Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental	One way to work out TO ÷ O mentally is to partition TO into a multiple of the divisor plus the remaining ones, then divide each part separately. Informal recording in Year 4 for 84 ÷ 7 might be: 84 70 + 14 ↓ ↓ ÷ 7 10 + 2 = 12 In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then

Method	Example
methods of multiplication, have not in the past been given enough attention. • Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.	each part is divided separately using the distributive law. Another way to record is in a grid, with links to the grid method of multiplication.  As the mental method is recorded, ask: 'How many sevens in seventy?' and: 'How many sevens in fourteen?'
Stage 2: Short division of T0 ÷ 20	
• 'Short' division of TO ÷ O can be introduced as a more compact recording of the mental method of partitioning. • Short division of two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. • For most children this will be in Year 4 (non-statutory) and Year 5 (statutory). • The accompanying patter is 'How many threes divide into 80 so that the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21 divided by three?' which gives the answer 7.	For 81 ÷ 3, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60 + 21. Each number is then divided by 3. The short division method can be shown and recorded like this: $3\overline{)60+21}$ (20 + 7) This is then progressed further and shortened to: $3\overline{)8^{2}1}$ (27) The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript. The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones.
Stage 3: Short division of HTO ÷ O	
• 'Short' division of HTO ÷ O can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction. • The accompanying pattern is 'How many threes in 290?' (the answer must be a multiple of 10). This	For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split the dividend of 291 into 270 + 21. Each part is then divided by 3. $291 \div 3 = (270 + 21) \div 3$ $= (270 \div 3) + (21 \div 3)$ $= 90 + 7$ $= 97$

Method	Example
gives 90threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7. • Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. • For most children this will be at the end of Year 5 or the beginning of Year 6.	The short division method is recorded like this: $3\overline{)290+1} = 3\overline{)270+21}^{\,90+7}$ This is then shortened to: $3\overline{)2\,9^{2}1}^{\,9\,7}$
Stage 4: Long division	
The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. The layout on the right, which links to the 'long division' method. Step 1 Divide- divide 15 into 4-which doesn't work. Divide 15 into 43 which equals 2. • Step 2 Multiply-multiply 15 by 2 which equals 30. • Step 3 Subtract-subtract 30 from 43 which equals 13. • Step 4 Bring down bring down the two to make 132. • Repeat the steps • Step 1 Divide- divide 15 into 132 which equals 8 • Step 2 Multiply-multiply 15 by 8 which equals 120 • Step 3 Subtract-subtract 120 from 132 which equals 12 There would be no more steps as 12 is smaller than 15 and would therefore be	How many packs of 15 can we make from 432 biscuits? Start by multiplying 24 by 1, 2, 3, 4, 5, and 10-you may need to add to your facts during your calculation. We use the acronym 'Do McDonald's Serve Burgers?' to help children remember correct steps to the question. $\begin{array}{rr\|ll} & & 2 & 8 \\ 1 & 5 & 4 & 3 & 2 \\ & & 3 & 0 & \downarrow \\ \hline & & 1 & 3 & 2 \\ & & 1 & 2 & 0 \\ \hline & & & 1 & 2 \end{array}$ Answer: 28 r12 = 28 12/15 = 28 4/5

Method	Example
our remainder. This would give us an answer of: 28 r12-in year 6 children have to be able to present their answers as a fraction. This would mean our remainder of 12 would become 12/15 as 15 was our divisor. In year 6 children also have to simplify answers and as 12 and 15 are both divisible by three the answer would therefore become 28 4/5	
For Years 5 and 6, children may use remainders expressed as fractions (in their simplest form) or decimals	456 ÷ 5 = 91 ⅕ Or 456 ÷ 5 = 91.2 091.2 5 ⟌ 4456.10